Contemporary Challenges in Mathematical Finance

Chris Kenyon

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UNIVERSITY of GREENWICH

Acknowledgements & Disclaimers

Joint work with Andrew Green, Chris Dennis, Mourad Berrahoui.

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Lloyds Banking Group: Postgraduate Entry Scheme

- Post-PhD, e.g. students just finishing doctorates
- Scope is Financial Markets, within Commercial Banking
- Plan to start scheme Sept 2017

Outline of the Presentation

Introduction

Introduction

- Mathematical Finance is about risk and reward, not mathematics
- Boundaries of material effects on pricing and risk management much better appreciated
- Dramatic growth of mathematical finance post-crisis
- Aim to give a flavour of post-crisis Mathematical Finance

Finance vs Mathematics: Self-Financing Portfolios 1/2

The Issue The value Y_t of a portfolio (using notation as (Duffie 2001)) composed of stock S_t and bond β_t with holding a_t and b_t can be written (Equation 14 on page 90):

$$Y_t = a_t S_t + b_t \beta_t$$

the change in portfolio value, or *gain process* is given as (Equation 15 on page 90):

$$dY_t = a_t dS_t + b_t d\beta_t$$

Clearly, if a_t is a delta hedge, i.e. a function of S_t, then applying the Itô-Döblin Lemma to the equation for Y_t would give:

$$dY_t = a_t dS_t + S_t da_t + da_t dS_t + b_t d\beta_t + \beta_t db_t + db_t d\beta_t$$

the extra terms are simply a mathematical consequence of applying the Lemma. This is the crux of this issue at the intersection between stochastic calculus (the Itô-Döblin Lemma) and finance (Duffie's equation 15), i.e. the concept of a self-financing portfolio.

Finance vs Mathematics: Self-Financing Portfolios 2/2

The Resolution is simply the *definitions* in (Harrison and Kreps 1979; Harrison and Pliska 1981) and reproduced in (Duffie 2001) that a self-financing portfolio follows (page 89):

$$a_t S_t + b_t \beta_t = a_0 S_0 + b_0 \beta_0 + \int_0^t a_u dS_u + \int_0^t b_t d\beta_u \quad (1)$$

or

$$d(a_t S_t + b_t \beta_t) = a_t dS_t + b_t d\beta_t \quad (2)$$

The only change in portfolio value comes from the value of the stock and bond (or cash account), whatever the trading strategy. The trading strategy can move value between the stock and cash accounts but not create or destroy value. If this were not true then the basic result that all self-financing portfolios have the same rate of return in the risk-neutral measure would be false (Harrison and Pliska 1981).

By definition of self-financing the only change in portfolio value comes from the value of the underlyings (the gain process). An additional self-financing equation is *implied*, here $S_t da_t + da_t dS_t + \beta_t db_t + db_t d\beta_t \equiv 0$, but it adds nothing since it is simply a direct consequence of the *definition* of self-financing.

New Context for Math Finance

New Context for Math Finance

- **Risk** is taken for **reward**
- **Regulations** require **capital** where there is risk of **loss**
- Regulations require **buffers** where there is risk of lack of **funding**

Regulations

Why do we care about financial regulations?

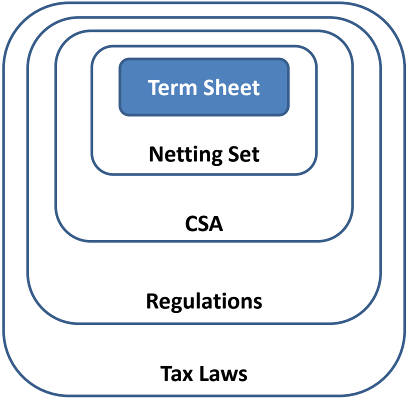
- Financial regulations are *designed* to change behaviour.
- Change what is permitted: constraints
  - in the US proprietary trading is forbidden for banks
  - standard swaps must now be cleared
- Change financial incentives, i.e. prices and costs.
  - Prices change market sizes.
  - Costs change market participants, incentivise internal reorganization
  - Prices and costs inform decisions on market entry or exit
- If regulations did *not* change prices, costs, and constraints then they would be pointless.

Financial regulations change valuations

Regulations

- How do regulations change valuations?
  - Capital requirements: e.g. Market Risk; Credit Risk; CVA Risk; Leverage Ratio
  - Funding requirements: e.g. Collateralization; Initial Margin; Liquidity Coverage Ratio; Net Stable Funding Ratio
- Are these free?
- How do they change valuation?
- Which valuation? Desk PnL; Accounting; Regulatory Capital?

Pricing boundaries have widened



Economic context has changed

Pre-Crisis, 2006

- Complete markets
- Perfect execution
- Zero profit and loss under all states of the world
- No funding costs
- No capital costs
- Single-curve pricing
- Spot risk analysis

Post-Crisis, 2016

- Funding costs, FVA (especially since 2008)
- Multi-curve pricing
- CSA discounting
- Lifetime regulatory costs
  - Capital (KVA) costs present since 2008, only formalized in 2014
  - Funding (MVA) costs more in focus via Bilateral IM and LR
- Unhedged position pricing
  - Open risk, so pricing measure in focus $\mathbb{P}, \mathbb{Q}, \mathbb{A}$
  - Market risk hedging: so multi-CSA discounting

Risk, mitigation and reward now part of pricing

Risks

- Market
- Credit
- CVA VaR
- Funding

Mitigation

- Back-to-back trades
- CDS, index-CDS
- Initial margin
- Capital
- Trade compression
- Trade re-couponing or resetting (e.g. re-setting cross-currency swaps)
- Settlement

Reward

- Profit
- Desk budgets

Theoretical context has widened

Theorem

If each market participant has different idiosyncratic continuous dividends when holding the same stock then there is no market-wide risk-neutral measure.

Proof.

Obvious. Let the stock price, from the point of view of market participant i, be:

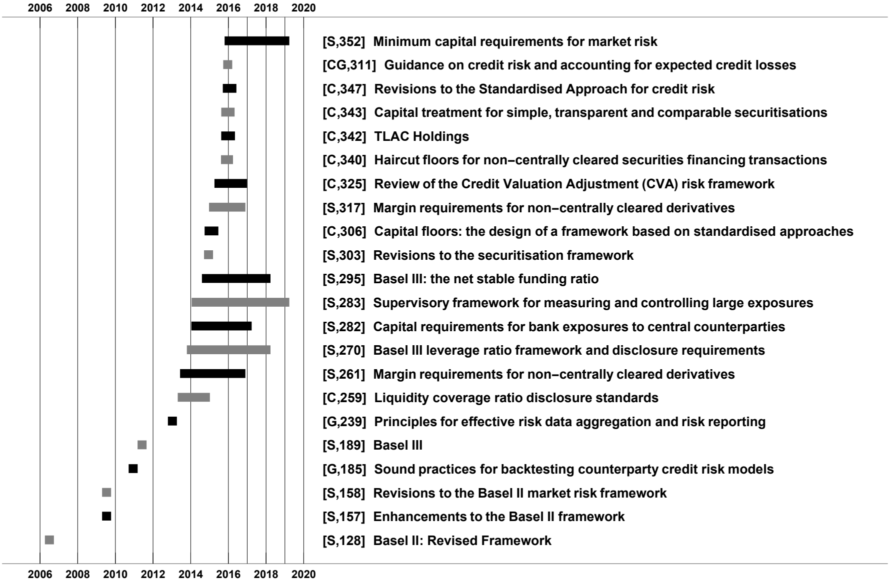
$$dS_i(t) = (\mu_i + a_i)S_i(t)dt + \sigma S_i(t)dW^{\mathbb{P}_i}(t)$$

where a_i is the objective dividend received by market participant i, and μ_i is the $\mathbb{P}$ drift believed by market participant i. This implies that in the idiosyncratic risk-neutral measure of i, the evolution of the stock price is:

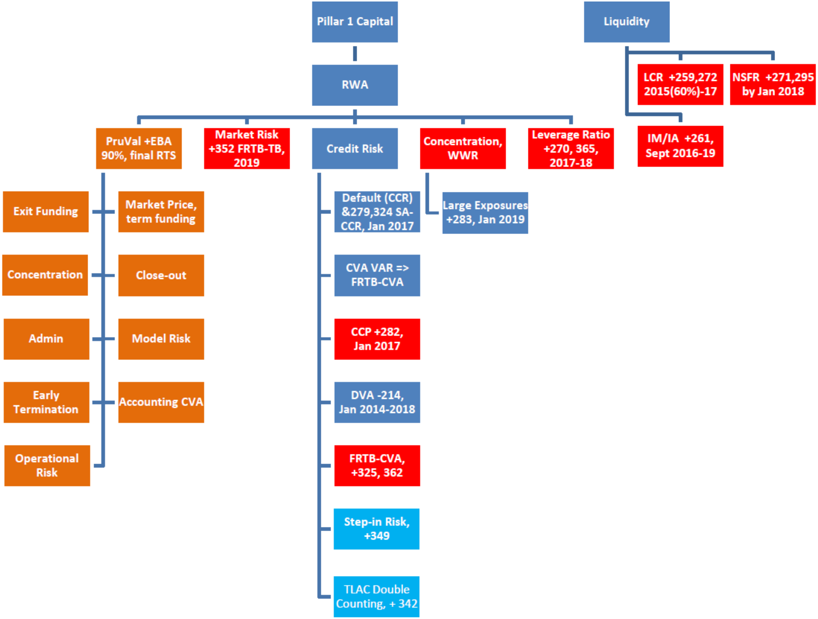
$$dS_i(t) = (r + a_i)S_i(t)dt + \sigma S_i(t)dW^{\mathbb{Q}_i}(t)$$

where r is the riskless rate. The $\mathbb{P}$ drifts of the market participants have been replaced by the riskless rate, but dividends are unchanged because they are objective although idiosyncratic. Hence there is no risk neutral measure because the rates of return are different for each participant (under each participants' risk neutral measures). □

Regulatory peak?



Regulatory landscape snapshot



Many lifetime costs have been formalized

					Bilateral			SOTA = state of the art			
Area	**VA**	**Source**	**Timing**	**Cleared**	**Subject to Bilateral IM**	**Collateralized, no Bilateral IM**	**Un-Collateralized**	**SOTA**	**Scope**	**Compute Unit**	**Notes**
Institutional Costs	OVA	Staff, facilities = cost / income ratio	Present	Y	Y	Y	Y	Beyond	Bank	C/P [2]	Overhead Valuation Adjustment
Bank Levy	BLVA	Liabilities, UK Gov.	Present	Y	Y	Y	Y	Beyond	Bank	C/P [2]	Turns into an extra tax 2016-2020
Credit	CVA	C/P Default	Present	minor	minor	minor	Y	Yes	C/P	C/P	Always get some, e.g. gap or haircut
Debit	DVA	Own Default	Present	minor	minor	minor	Y	Debateable	Bank	C/P [1]	Discredited, removed from US FASB, and capital
Funding	MVA	Initial Margin (VaR/ES)	Present & Sept 2016	Y	Y	contingent	contingent	Yes	C/P [1]	C/P [1]	Largest banks hit first
Funding	MVA	Concentration, Default Fund, Gamma	Present & TBD	Y	TBD	N	N	Beyond	C/P [1]	C/P [1]	Concentration and other items are TBD in SIMM
Funding	FVA	Variation margin (part)	Present	Y	Y	Y	N	Yes	C/P [1]	C/P [1]	Cost and benefit
Funding	ColVA	Variation margin (part)	Present	Y	Y	Y	N	Yes	C/P [1]	C/P [1]	Depends on collateral rate
Funding	SVA	hedging Strategy	Implicit	N	contingent	contingent	contingent	On Horizon	Bank	C/P [1]	Implicit (observed) from hedging strategy
Funding	LCRVA	Liquidity buffers (LCR)	Present	Y	Y	Y	minor	On Horizon	Bank	C/P [1]	Basel III; PRA version already present (will transition)
Funding	DTVA	Liquidity buffers (Downgrade Triggers)	Present	N	N	contingent	contingent	Beyond	Bank	C/P [1]	PRA liquidity buffers on downgrade (3 notch requirement)
Funding	NSFRVA	Liquidity ratio barrier (NSFR)	2018	Y	Y	Y	Y	On Horizon	Bank	C/P [2]	Basel III
Capital	PVA	Prudent Valuation	Soon	Y	Y	Y	Y	Yes	Bank	Various [3]	Enters into effect on publication
Capital	KVA	Market Risk	Present	Y	Y	Y	Y	Beyond	Bank	C/P [2]	Basel III, FRTB updates. Modelling of future MR unclear.
Capital	KVA	Counterparty Credit Risk (including 2% from CCPs)	Present	tiny	tiny	tiny	Y	Yes	C/P	C/P	Always get some, e.g. gap or haircut
Capital	KVA	CVA variation capital	Many EU exemptions	N	tiny	tiny	Y	Yes	C/P	C/P	Basel III, FRTB-CVA updates. EBA working to remove exemptions. Always get some, e.g. gap or haircut
Capital	KVA	Leverage Ratio	Present	Y	Y	Y	Y	On Horizon	Bank	C/P [2]	Basel III, Dodd-Frank (tbd)
Capital	KVA	CCP capital (DF)	with FRTB	Y	N	N	N	On Horizon	C/P	C/P	Basel III
Tax	TVA	Tax	Present	Y	Y	Y	Y	On Horizon	Bank	C/P	Consequence of profit paying for capital and non-perfect hedging. Bank Levy becomes fully a tax in 2020.
Legal	LVA	Legal	Present	Y	Y	Y	Y	Beyond	C/P	C/P	Difference between Legal claim recovery and economics.
Settlement or Reference	Base	BSM, Piterbarg	Present	Y	Y	Y	Y	Beyond	Both	C/P	Consequence of moving to settlement avoiding addon in LR capital costs

[1] if structurally short funding
[2] assuming *policy-based* ratio management
[3] reported to be small because effects controlled as discovered

Key post-crisis technical steps formalize economics

- Post-crisis world starting to accept that profit and loss exist in theoretical approach to pricing
- Key technical steps:
  - Multi-curve pricing (Mercurio 2010a; Mercurio 2010b; Kenyon 2010; Moreni and Pallavicini 2014)
  - CSA pricing (Piterbarg 2010; Piterbarg 2012)
  - No self-hedge + funding strategy: Semi-replication (Burgard and Kjaer 2012)
  - Capital (Green, Kenyon, and Dennis 2014)
  - Initial Margin (Green and Kenyon 2015)
  - Uncollateralized counterparties not credit hedged: Double-semi-replication (Kenyon and Green 2015)
  - Reward, open risk pricing in Bank Risk Appetite measure $\mathbb{A}$: (Kenyon, Green, and Berrahoui 2015)
  - Multi-CSA pricing for uncollateralized counterparties: (Kenyon and Green 2016)
  - Change in collateralized counterparties not hedged: Triple-semi-replication (Kenyon and Green 2016)

Funding Regulations

Funding Regulations: Initial Margin

- Motivation for Initial Margin (aka Dynamic Initial Margin)
- Sources
  - BCBS/IOSCO (BCBS-261 2013; BCBS-317 2015): 99% one-sided 10-day VaR calibrated to historical period of significant financial stress (or a schedule)
  - CCP methods are proprietary but can reasonably be assumed roughly similar to, but not identical to BCBS/IOSCO
- Implementations
  - US requirements, final rule Oct 30, 2015, entry into force Sept 2016 (FDIC 2015)
  - EU requirements, final draft RTS March 8, 2016 entry into force Sept 2016 (JC-2016-18) ... or later (Bloomberg and WSJ reports)
  - ISDA SIMM(TM) is proprietary, US patent applied for (62/154,261), **licensing now required** for use (April 5, 2016) unlike previous announcement (June 1, 2015), current version (ISDA-SIMM-3.15 2016) is incomplete (concentration calibration missing)
  - Other
- Effects, e.g. IM currently contributes to the Leverage Ratio, see footnote 12 in (BCBS-270 2014), but does not do so in (BCBS-365 2016)
- Lifetime costs: (Green and Kenyon 2015) extends (Burgard and Kjaer 2013) semi-replication to cover economics of lifetime costs of initial margin (MVA)

BCBS/IOSCO Initial Margin: Motivation

- "Limit excessive and opaque risk-taking through OTC derivatives and to mitigate the systemic risk posed by OTC derivatives transactions, markets, and practices."
- Agreed in principle 2011 by G20 and delegated to BCBS/IOSCO
- Objectives
  - Reduction of systemic risk
  - Promotion of central clearing
- Not clear that promotion of central clearing reduces systemic risk when there are only a 4 or 5 major CCPs
- Not clear that switching from credit risk to liquidity risk reduces systemic risk
- An effect like a financial transaction tax may indeed reduce market activity and hence may reduce systemic risk. Previously this was done by capital but now joined by margin for margined transactions.

Margin vs Capital

- Capital is not seen as protection from systemic shock
  - “capital is shared collectively by all the entitys activities and may thus be more easily depleted at a time of stress”.
  - “Capital requirements against each exposure are not designed to cover the loss on the default of the counterparty but rather the probability-weighted loss given such default”
- Margin is designed to cover whatever losses are in scope: “targeted and dynamic”
  - “each portfolio having its own designated margin for absorbing the potential losses in relation to that particular portfolio,”
  - “margin is defaulter-pay“
- Margin is “defaulter pay” **and** “survivor-pay” for uncollateralized clients. Logic in document is incomplete.
- **Someone will always pay, and it will not be the defaulter — because if the defaulter could pay they would not default**. Only choice is who gets hit. Here major financials are protected at the expense of bank clients. Main protection is reduction in activity.

Scope

- All financials and major non financials: $\geq$ EUR8B in gross notional
- Exceptions entities:
  - Sovereigns
  - Central banks, multilateral development banks, BIS
- Exceptions products:
  - Physically settled FX forwards and swaps
  - Principal exchange part of cross currency swaps
  - EU Covered bond pools
  - EU Single stock options and equity index options: delayed implementation
- Threshold: EUR 50M
- MTA: EUR 0.5M

IM Definition

- 99%, 10-day, one-tailed confidence level (i.e. VaR) based on data including significant period of stress, and calculated per legally enforceable netting set

EU MPOR is at least 10 days, and is increased to cover liquidity, volume, and number of participants

- Stress period separate for each major asset class. EU classes:
  - interest rates, currency and inflation;
  - equity;
  - credit;
  - commodities and gold;
  - other.

EU Data 3Y to 5Y, at least 25% from stress period: replaces oldest data if not in most recent continuous data

- Recalibrate every 12M
- Internal models require supervisory approval, internal governance, and continuous assessment

IM Collateral

- Exchanged gross
- Must be segregated and bankruptcy-remote
- Only Buy-Side can rehypothecate, and with limitations

EU can be almost anything, with some quality limits and haircuts

EU limits w.r.t. wrong way risk

EU concentration limits

- Excluded from Leverage Ratio capital (BCBS-365 2016)

Capital Regulations

Introduction to Capital

Objectives

- This section gives a brief introduction to Capital in Banking
- Provides an overview of the key concepts
- Explores traditional approaches to capital pricing and their links to derivative pricing

Capital and funding costs drive bank re-design 2008—

- Financial crisis of 2007-8 appeared as a liquidity crisis driven by a credit crisis
  - Liquidity crisis = inability of institutions to access funding at competitive prices
  - Credit crisis = uncertainty on solvency of financial institutions
  - Two large investment banks[1] converted to Bank Holding Companies to access government funding
- Regulators have addressed both liquidity and solvency (BCBS-189 2011; Dodd and Frank 2010)
  - Liquidity via LCR and NSFR
  - Solvency via increased capital
  - McKinsey (2102) calculated that RoE declined on average 65% from 20% to 7% RoE with only FX and Cash Equities remaining above 10% (16% and 15% respectively).
- Banks have reorganized, and continue to reorganize as regulations come into effect

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[1] http://www.goldmansachs.com/media-relations/press-releases/archived/2008/bank-holding-co.html, http://www.morganstanley.com/about-us-articles/6933.html

Capital

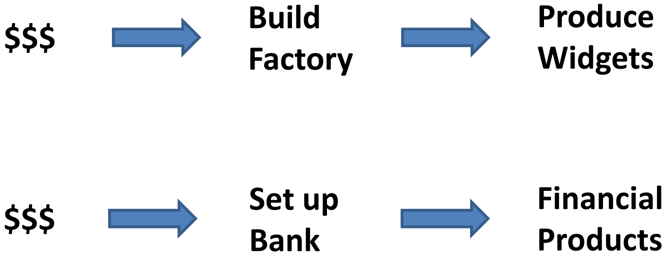
- What is capital?

  "That part of a man's stock which he expects to afford him revenue is called his capital."
  Adam Smith

- Mix of debt and equity that finance a firm (Brealey, Myers, and Allen 2010)

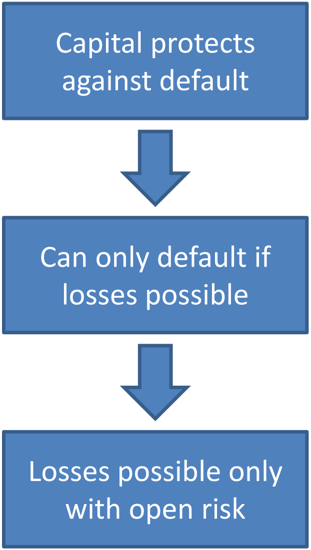
- Capital supply and demand
  - Regulatory Capital (BCBS-189 2011), defines both supply of capital and demand for it
  - Economic Capital Models: internal bank models for may cover both supply and demand
  - Accounting: details in last Section
- Economic and Regulatory models do not have to agree, but Regulatory model sets floor for demand and ceiling for supply, with consequences for breaches

Capital enables production



- Capital ($$$) is in use, e.g. wherever funding is used

Capital mitigates risk



Capital and Corporate Finance

- Corporate Finance deals with capital as one of the aspects of setting up, running, and closing down a company. Standard texts include (Brealey, Myers, and Allen 2010)
- Mathematical Finance is a specialised subset of Corporate Finance
- Object of capital budgeting is to find assets that are worth more to the firm than their cost
- Objective of Corporate Finance is to maximize the current market value of the firm's outstanding shares

Opportunity Cost of Capital

Opportunity cost of capital is standard in Corporate Finance for comparing equivalent-risk projects

- Why? Because capital budgeting is about finding assets that are worth more than their cost, because this maximizes current market value of the firm's shares
- Suppose that you can borrow at 5%, does this make 5%, the cost of capital for project X? Suppose now that
  - Project X has a return of 10%
  - An equally risky project Y has a return of 15%
- The opportunity cost of capital to project X is 15%, not 5%

Many caveats to this analysis for example

- requires equivalent risks that are accessible
- no comments on constraints
- no comments on capital raising vs allocation vs returning to shareholders

NPV is the standard decision tool

Weighted Average Cost of Capital

- Cost of capital for a firm is the cost of funds across both debt and equity
- The Weighted Average Cost of Capital (WACC)

$$\text{WACC} := \frac{\sum_{i=1}^{N} r_i V_i (1 - t_i)}{\sum_{i=1}^{N} V_i} \tag{3}$$

  r_i required rate of return for security i,
  V_i market value,
  t_i tax rate,
  there are N forms of capital.

- Typically beneficial from a tax perspective to issue debt rather than equity
- The mix of capital instruments gives the capital structure. The investor rights associated with each tier are different.

Modigliani-Miller and Kenyon-Green

- Modigliani-Miller theorem (Modigliani and Miller 1958) states that the value of a firm is independent of the debt-equity mix with which it is funded, assuming that there is no interaction between the firm and its funding
- Kenyon-Green theorem (Kenyon and Green 2014) states that if different market participants have different holding costs for the same asset then there is no market-wide risk-neutral measure that is valid for all participants
- The two theorems address different, but complementary, questions
- Kenyon-Green states that if the value of the project depends on the firm then the value of the firm (as a collection of projects) is not independent of its funding because the funding level required will be different for different firms
  - In Modigliani-Miller terms, the **same** project done by different firms is actually **not the same** because it interacts with the firm, e.g. its Regulatory status

Capital Asset Pricing Model (CAPM)

- William F. Sharp (Sharp 1964) was included in the 1990 Nobel Prize in Economics for CAPM
- CAPM gives the following relationship for the expected return on an asset i

$$\mathbb{E}[r_i] = r + \beta_i(\mathbb{E}[r_m] - r) \tag{4}$$

  r is the risk free rate, $\mathbb{E}[r_m]$ is the expected return of the market and

$$\beta_i = \frac{\mathrm{Cov}(r_i, r_m)}{\mathrm{Var}(r_m)} \tag{5}$$

  restating this we have that beta give the risk premium

$$\mathbb{E}[r_i] - r = \beta_i(\mathbb{E}[r_m] - r) \tag{6}$$

- based on Mean-Variance analysis of portfolio returns for which Harry Markowitz (Markowitz 1952a; Markowitz 1952b) was also included in the 1990 Nobel Prize in Economics

CAPM key points

- Deals with open risk
- Single-period, mean-variance portfolio optimization
- Does not include constraints — Danzig just missed the Nobel prize when he invented Linear Programming, Integer Programming, Stochastic Programming, etc., etc. which generalize CAPM for constraints, many periods, etc.
- Has had many modifications over the years to deal with cases where it is empirically false, e.g.

$$\begin{aligned}\mathbb{E}[r_i] = r &+ \beta(Market\ Price\ of\ Risk)\\ &+ (MarketPrice\ of\ Size\ Risk)\\ &+ (Market\ Price\ of\ Company(i)\ Specific\ Risk)\end{aligned}$$

  "the failure of the CAPM in empirical tests implies that most applications of the model are invalid" (Fama and French 2004). Eugene Fama was awarded the 2013 Nobel price for empirical analysis of asset prices
- CAPM is a stylized approximation, just like Black-Scholes-Merton. Good for intuition but **never use as-is**

Replication, Semi-, and Double-Semi-Replication

Replication (Black-Scholes-Merton 1973)

- Build a self-financing portfolio from assets with known prices that has exactly the same cashflows as the target under all states of the world
- By no-arbitrage cost at time zero of the assets you need is the cost of the target

Semi-Replication (Burgard and Kjaer 2013)

Do not hedge all cashflows on own default

Double-Semi-Replication (Kenyon and Green 2015)

Do not hedge all cashflows on own default and do not hedge all cashflows on counterparty default

- Still want to get paid for open risk — can use variants of CAPM for that, generally inspired by CAPM rather than anything so simplistic
- Incomplete market pricing

RAROC

- Risk-adjusted return on capital (RAROC) is a method of measuring financial performance adjusted by risk
- RAROC is defined by

$$\text{RAROC} = \frac{\text{Expected Return}}{\text{Economic Capital}} \tag{7}$$

- i.e. the return is weighted by the amount of economic capital required to cover the risk
- In trading context economic capital is often replaced by VAR measures
- RAROC is can be used to assign capital and make management decisions around businesses

Multi-period RAROC

- RAROC formula over N reporting periods:

$$\text{Margin} - \sum_{n=1}^{N} \frac{\text{Capital}_n - \text{Capital}_{n-1}}{(1 + \text{RORAC})^n} = 0$$

- Solve for RAROC and only do projects that are above a hurdle rate r_{hurdle}, say 10%

Multi-period RAROC vs KVA (1/2)

- Basic equation for KVA, where cost of capital $\gamma_K(t)$ is deterministic:

$$\text{KVA} = -\int_t^T \gamma_K(u) \exp\left(\int_t^u r(s) + \lambda_B(s) + \lambda_C(s) ds\right) \mathbb{E}_t[K(u)] du$$

- Now solving for Margin=KVA we have RAROC equal to the hurdle rate, so

$$\text{KVA} = \sum_{n=1}^{N} \frac{\text{Capital}_n - \text{Capital}_{n-1}}{(1 + r_{\text{hurdle}})^n}$$

Multi-period RAROC vs KVA (2/2)

- Moving the RAROC equation to continuous time and continuous compounding, and using $K(u)$ for Capital we get (since IRR has only one rate):

$$
\begin{aligned}
\mathrm{KVA} &= -\int_t^T r_{\text{hurdle}}\; e^{r_{\text{hurdle}}\;(u-t)}\mathbb{E}_t[K(u)]du && (8)\\
&= -\int_t^T \gamma_K(u) e^{-\int_t^u r(s)+\lambda_B(s)+\lambda_C(s)ds}\mathbb{E}_t[K(u)]du && (9)
\end{aligned}
$$

so

$$\gamma_K(u) = r_{\text{hurdle}}\; e^{r_{\text{hurdle}}\;(u-t)+\int_t^u r(s)+\lambda_B(s)+\lambda_C(s)ds}$$

- So we can move between KVA and RAROC viewpoints

Cost of Capital Discounting?

- A common approach to cost of capital calculations uses cost of capital discounting on future capital amounts
- Hidden assumption is that capital cashflows are going-to and coming-from a bank account that provides a interest rate equal to the cost of capital
  - Why privilege capital cash flows above other cash flows? How tell the difference?
  - Not supported by evidence: all cashflow for capital is one way, from the trade to the owners of the capital
  - Capital cashflows are rent
- KVA with open risk pricing is a more consistent approach

Pricing without Hedging

Open Risk

- Open risk, and hence limits, is a widespread feature of banking and one motivation for capital
- Pricing open risk, i.e. warehoused risk, is starting to attract attention (Hull, Sokol, and White 2014b; Kenyon and Green 2015)
- Long history in portfolio construction (Markowitz 1952a) and investment evaluation (Sharpe 1964).
- Valuation adjustments on prices for credit also have a long history (Green 2015) but only recently has capital has been incorporated (Green, Kenyon, and Dennis 2014).
- The contribution of this paper is to develop and propose a method of computing risk limits consistent with a bank's Risk Appetite Framework (RAF) (BCBS-328 2015; FSB 2013) using the Risk Appetite Measure $\mathbb{A}$, defined here.

Historical ($\mathbb{P}$) vs Risk-Neutral ($\mathbb{Q}$) Calibrations

- **Historical Calibration**
  - Calibrate to a historical period
  - Within **limits** stakeholders have significant flexibility
- **Risk-Neutral Calibration**
  - Risk-neutral pricing provides prices at $t = 0$
    - Binary options on portfolio value give discounted-probability distribution
  - Inverse of market price of unit payment gives PFE
  - Measure-independent if market is complete
- **Mixed**
  - Risk-Neutral for market observables
  - Historical for non-observables, e.g. correlations
  - Often pick-and choose approach by stakeholders

Historical vs Risk-Neutral Calibrations

- Difference can be described by the price of risk

Price of Risk for Historical Calibration

- In the usual Black-Scholes-Merton setup

$$m_M \equiv \frac{\mu - r}{\sigma}$$

  m_M is defined as the market price of risk (confusing terminology!)
- The **riskless** rate of return is r
- The rate of return on **open risk** is μ
- The risk is the volatility of the return, i.e. σ

Price of Risk for Risk-Neutral Calibration

$$m_{\text{RN}} \equiv 0$$

The Price of Risk

- How should open risk be priced? Standard answers:
  - Assume there is no systematic risk and hence have price open risk at zero cost: market-implied pricing
  - Mean-variance hedging as in (Markowitz 1952a), and modern versions described in (Birge and Louveaux 2011)
  - Look at the real world and price accordingly
- Limitations:
  - Is there really no systematic risk?
  - Does mean-variance align with institution's perception of risk?
  - How do you calibrate a real world measure?
  - For IMM banks risk factor dynamics must pass historical backtesting
- Essentially we are discussing the price of risk
  - Widely discussed in academic literature as the *market price of risk* (Berg 2010; Hull, Sokol, and White 2014a)
  - Will have a term structure (Hull, Sokol, and White 2014a)
  - Can have different prices for different risks (drift, volatility, correlation, etc.)

Market's Price of Risk can be observed, in theory

Market Price of Risk

- In the usual Black-Scholes-Merton setup

$$m_r^M \equiv \frac{\mu - r}{\sigma}$$

  m_r^M is defined as the price of risk according to the market (M)
- The **riskless** rate of return is r
- The rate of return on **open risk** is μ
- The risk is the volatility of the return, i.e. σ

- The size of the literature shows how subjective the market price of risk is in practice
- Is this relevant anyway?

What is the Bank's appetite for risk?

Risk is taken for reward

- Definition
  "Risk appetite is generally expressed through both quantitative and qualitative means and should consider extreme conditions, events, and outcomes. In addition, risk appetite should reflect potential impact on earnings, capital, and funding/liquidity."
  (Senior Supervisors Group, Observations on Developments in Risk Appetite Frameworks and IT Infrastructure, December 23, 2010)
  - Observed in action — not a single utility function.
  - No requirement for a coherent (Delbaen 2000) definition
  - Part of Basel II with renewed emphasis post-crisis
- Metrics and controls **already in place** describe the appetite that a bank has for risk. **Observable.**

Bank's price of risk can be observed defines the Risk Appetite Measure (RAM)

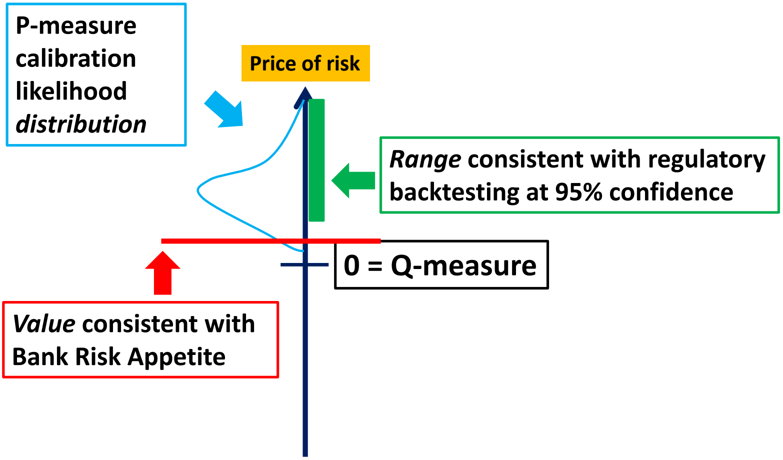
Bank Price of Interest Rate Risk

$$m_B = \frac{\left(\frac{\text{Rates Desk Budget}}{\text{Rates Desk Investment}} - 1\right) - r}{\sigma_{\text{Rates VaR}}}$$

- r is riskless rate
- $\sigma_{\text{Rates VaR}}$ is the implied volatility from the Rates VaR limit

- Economically profits can come from two sources
  - rents, e.g. from monopoly position
  - risk taking
- Consider post-rent profits under normal competition

Prices of Risk: given a Bank's Risk Appetite it has *already* chosen a price of risk



This defines **Risk Appetite Measure** (RAM). Complements $\mathbb{P}$, $\mathbb{Q}$ calibrations

Price of risk from risk appetite: Normal Model

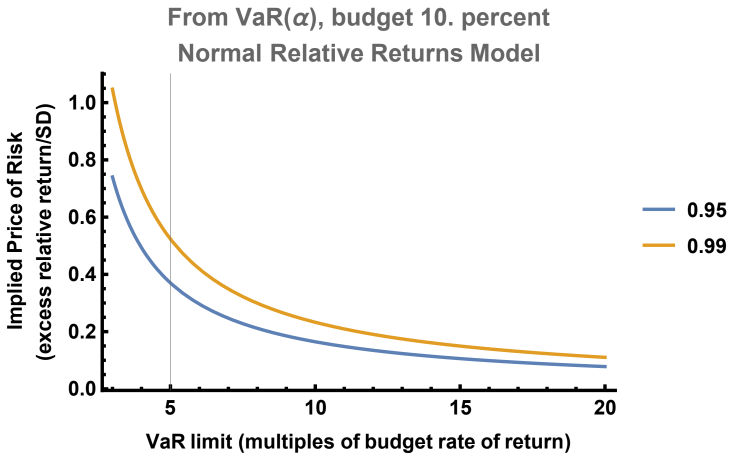
- Assume that relative returns follow Normal distribution
- Fix desk budget rate of return, μ
- Fix desk limit VaR(q) in units of desk budget rate of return, L

$$\sigma_{\Rightarrow} = \frac{\mu(1-L)}{\sqrt{2}\,\mathrm{erfc}^{-1}(2q)} \tag{10}$$

$$m_B = \frac{\mu - r}{\sigma_{\Rightarrow}} \tag{11}$$

- Normal distribution has two parameters, so two constraints, desk budget and VaR limit, are sufficient

Implied Bank Price of Risk



Conclusions

Conclusions

- Mathematical Finance is growing rapidly **away** from its traditional core
- **Realism** (risk/reward) and **transparency** (XVA) are key
- **Computational challenge is extraordinary:**
  - First generation XVA (CVA, FVA) need simulation of future portfolio values
  - Second generation XVA (KVA, MVA) need simulation of future portfolio (all netting set, and all trade) sensitivities
  - Need to manage risk on XVA

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