Tomorrow's Mathematicians Today

an Undergraduate Mathematics Conference
supported by the IMA
University of Greenwich
13th February 2016

Abstracts

Contents

The student speakers' abstracts are presented in the order in which the talks are listed in the conference programme. Since the abstracts were submitted in a variety of formats, not all pages are numbered.

Keynote

Dr. Eugenia Cheng
University of Sheffield and Scientist in Residence at the Art Institute of Chicago

Deep Structures: my journey into abstract mathematics

I will talk about mathematics as a way of thinking, as opposed to a way of calculating the correct answers. I will give a personal account of how I was drawn to more and more abstract mathematics, until I finally found my mathematical home in Category Theory. Abstraction for me is the process of shedding details to get to deep structures that are holding things together. I will describe some of my favourite abstract structures and show how abstraction reveals connections between apparently unrelated ideas.

Andrew Silverman, Birkbeck, University of London

Pascal's Triangle: Beyond the Non-Negative Integers

Pascal's triangle takes its name from the French mathematician Blaise Pascal, who published his "Treatise on the Arithmetical Triangle" in 1653, although it had already been known about for centuries in other parts of the world. It is a triangular array of the binomial coefficients and as such contains only non-negative integers, with the rows and columns also corresponding only to non-negative integers.

This talk will focus on attempts to extend Pascal's triangle beyond the non-negative integers to the real numbers, and yet further to the complex numbers. We will try to visualise these extensions where possible and in doing so will encounter the idea of 'Pascal's surface'.

Max Masters, University of Greenwich

How I scientifically lost my money on the Grand National

“Could you apply it to horse racing?” asked the Police Inspector in the front of the car. I'd spent the last few minutes explaining how bookmakers tended to be more accurate predicting winners of basketball games which are high scoring compared to low scoring games like football. I'd been reading *The numbers game* (Chris Anderson and David Sally), and found it inspiring.

I took on a challenge by Branded3 to create a formula for a perfect bet on 2015s Grand national. I based my model around the weighting of the factors surrounding performance of horses. The idea stemmed from an attempt to rank baseball players (yep you guessed it, from *Moneyball*). Each factor contributes to an individual’s overall score, the idea being that the model is then dynamic and can be altered to reflect the conditions that occur.

My model took me to being published in three national newspapers and earned me a free £50 bet on my chosen horse. I'll be explaining my model, its factors and the ideas that inspired it.

Oh and the Police Inspector? That would be my father driving
.

Rachael Whyman, University of Kent

Tiling Time with Music and Juggling

In this talk I will compare two real-life applications of mathematics: musical tilings and juggling patterns. In its simplest form, musical tiling is using a number of voices, each performing a rhythmic pattern that, when performed together, fill the musical line/time axis. Juggling diagrams split up the timeline into beats, with each juggling ball's pattern being represented by a distinct line, which is connected to each beat the ball is thrown/caught on. It was only in the mid-1980's that mathematicians began developing a formal language to express juggling patterns. The first mathematical study of Musical Rhythmic Tilings was more recent still, being Dan Tudor Vuza's papers beginning in 1991. My key observation is that both musical tilings and juggling patterns seek to achieve the same result: a tiling of the time axis. This implies that there could be an equivalence. I will outline the similarities, define a mapping from one set onto the other, and show which properties the mapping respects. As I consequence, I will explain when results in juggling can have analogues in musical rhythmic tilings. I will conclude by explaining how the shared properties and analogues between the two sets has increased our understanding of both areas.

From Butterflies to Bridges: A Study of Wave Propagation Through Periodic Structures

Tom O'Neill, Aberystwyth University

We see periodic structures everywhere, from the wings of a butterfly to phenomena such as the wobbling that was seen in the Millennium Bridge in London in the early 2000s. In this presentation, we aim to understand the links between these structures and how we can quantify their behaviour.

We begin with a discussion on how periodic structures appear in nature, looking particularly at iridescence and metamaterials. We then start to look at some of the classical theory developed in the late 19th and early 20th centuries, looking at discrete periodic structures, before using more modern, and more advanced mathematical tools, such as the Dirac Mass and the Green's function of a system, to obtain the solution for the modelling equation of a system.

For the rest of the talk, we will use the methods that we have discussed to solve a more concrete example. We will make some modelling assumptions and then use the equation of motion, combined with the techniques we establish here, to find the Green's Function for this system. We conclude by comparing the amplitude profiles of two different bridge models and see which holds up best under a given load.

The Story of Émilie du Châtelet

Georgia Rubinstein, University of East Anglia

Whilst writing my final year dissertation on 'The Spread of Calculus', I discovered the mathematician Émilie Du Châtelet. Her talent and contribution to the world of mathematics astounded me, yet there are only traces of her success throughout history of mathematics literature. Born into a French society where women were perceived as merely objects rather than people with aspirations and interests in education, Émilie was not encouraged to pursue a mathematical life. To fill this void she desired an intellectual partner to work with and the famous Voltaire was the perfect fit. Nurtured by his prestige in the literary world, Émilie focused her attention to the mathematics of the universe. Here she encountered the works of Isaac Newton and as a result he became a strong influence on her work. Subsequently, she translated his 1731 third edition of the *Principia* into French. This became her most celebrated achievement and is the most famous translation used today. Émilie's edition contains her own commentary of Newton's work, which demonstrates her stunning understanding of his calculus, which left the most able mathematicians at the time bewildered. Émilie's influence to the spread of calculus is extremely underrated; she enriched mathematics with her precision, detail and clarity. To me, Émilie Du Châtelet is a truly brilliant mathematician who deserves more credit and recognition.

Abstract

Mathematical Models of Growth Factor Activation during Bronchoconstriction in Asthma

Hannah Pybus, Supervised by Dr. B. Brook

2nd December 2015

Airway hyper-responsiveness, chronic inflammation and remodelling characterise asthma. It is possible that airway contraction may activate Transforming Growth Factor $\beta 1$ (TGF-$\beta 1$), thus triggering excessive responses that contribute to airway remodelling. Effects of cell-mediated TGF-$\beta 1$ activation, during an asthma attack, continue to be debated.

The aim is to understand why airway smooth muscle cell contraction induces increased TGF-$\beta 1$ production, yet reduces TGF-$\beta 1$ production when stretched. Informed by ongoing lung slice experiments undertaken in Dr Tatlers QMC laboratory, the primary focus is to produce a mathematical model closely resembling the experimentally observed behaviour.

We have developed a system of ODEs and identify the dominating parameters using a multiple timescale analysis. Through successful linear stability and characteristic polynomial discriminant analysis we have identified critical qualitative bifurcation points. These yield potential parameter thresholds for observable asthmatic behaviour. Thereafter, we refine our ODE model to include spatial variation, proposing a system of PDEs and incorporate breathing patterns in the form of oscillatory strains. In this context we will investigate diffusion driven instability and subsequent pattern formation. We will extend our one-dimensional model to the circular geometry of a contracting airway, which will provide insight into mechanisms that may operate in the intact airway *in vivo*.

Demetra Vasiliou The University of Glasgow

Lagrange's Contribution to Mechanics

Abstract

This document is based on my 4H Project on Lagrange's Contribution to Mechanics. It's supposed to draw an outline on what is going to be presented at the conference in Greenwich on the 13th of February 2016. The topic of the project is the work done by Joseph-Louis Lagrange regarding the field of Mechanics. I will talk about the three eras his life can be divided into as well as about his achievements including his publications and letters which he wrote to other scientists and the work he did on the tautochrone and on the calculus of variations. Moreover, the presentation will include details on Lagrange's life, the Private Scientific Society which was founded by Lagrange and about the Scientific Journal which was published through that period which included a lot of papers and works of Lagrange. Furthermore, I will talk about the way Lagrange managed to gain a position in Prussia, how he managed to receive the offer from the French Government to be appointed a position at the Parisian Academy and details on the publication of his masterpiece, Mecanique Analitique and finally about Lagrangian Mechanics which will include all of Celestial mechanics, the Three-body problem and Lagrangian Points.

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Matjaz Leonardis, University of Oxford

What group theory is really about

Mathematics courses and textbooks traditionally present groups as algebras satisfying a list of three axioms. The choice of axioms is often presented as essentially arbitrary or else said to encompass a number of important mathematical structures. Yet examples of groups that are usually given - such as symmetries of solids or the additive algebra of real or integer numbers - while perhaps conceptually simple, still often seem to have very little to do with theorems that are subsequently discussed and leave many of them looking parochial or arbitrary.

However many of the ideas one would encounter in a first course in group theory can trace their origins back to the late 18th and early 19th century to the people who were intensely engaged with the problem of algebraic solutions of polynomial equations and the reasons for why such solutions exist or do not exist.

The talk will focus on several early group theoretic ideas and will show, in the context of that problem, that these ideas were far from arbitrary and that each was relevant to it in a quite substantial and wonderful way.

Andris Thompson, University of East Anglia

Applications of Mathematics to Lepidopterology: Chaos Theory and the Butterfly Effect

A general introduction to chaos theory, with a particular focus on unusual and unexpected applications to physical and biological systems. Necessary definitions and methods are given from first principles, for both continuous and discrete chaotic systems, including cobweb diagrams, phase plane analysis and strange attractors.

Once the general principles are known, practical examples – along with a brief note on historical context and motivations – are given, starting with the famous Lorenz equations and chaos in weather prediction. Various biological systems are discussed, starting with simple population dynamics models, and continuing on to patterns of extinction and complex, interlinked systems.

Finally, in light of the behaviour of real chaotic systems, the talk will conclude with a more fundamental, unifying idea of the true nature of chaos and predictability, plus current developments and possible future areas of research.

All are welcome, and no more than a basic understanding of calculus is required or expected.

Jake Holdroyd, Nottingham Trent University

Bayesian modelling of Platelet Aggregation

Blood clotting is an important process that is useful for healing wounds, but has a negative effect in deep vein thrombosis; this is why there is a need to understand the process. One key process involved in blood clotting is GPVI receptor binding, which allows cytosolic kinase to phosphorylate, enabling the receptor to send a signal downstream that triggers the clotting response. Dunster *et al.* (2015) used the law of mass action to describe this process via a number of ordinary differential equation (ODE) models, the simplest of which comprised seven ODEs with eight parameters. Dunster *et al* then used hypercube sampling combined with a gradient-descent algorithm to estimate the parameters and fit the model to the observed experimental data.

In this work, we adopt a Bayesian approach (unlike Dunster *et al)* to obtain the posterior distribution of one of the parameters in the model via Markov Chain Monte Carlo (MCMC). We then use the Metropolis Hastings algorithm to sample from its posterior distribution in order to produce a better estimate of the parameter. Finally, we plot a new estimated curve alongside the observed experimental data to visually compare our results with those of Dunster *et al*. To conclude we will show that using Bayesian methods we can potentially find more accurate estimates of parameters.

Dunster JL, Mazet F, Fry MJ, Gibbins JM & Tindall MJ (2015). Regulation of Early Steps of GPVI Signal Transduction by Phosphatases: A Systems Biology Approach. *PLoS Comput. Biol.* **11(11)**:e1004589. doi:10.1371/journal.pcbi.1004589

Beyond Fourier: A Study of Wavelets

Daniel Burrows

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Inspired by its power but frustrated by its limitations, 20th century mathematicians sought to utilise the algebraic formalism at the heart of Fourier analysis as a way of constructing more advanced methods of approximating square-integrable functions. Capable of analysing classes of functions that resisted the applications of Fourier analysis, the wavelet was born, and with its inception came numerous and significant advances in both science and mathematics.

For example, Paul Lévy, a French physicist in the 1930's, found that the Haar wavelet, defined by

$$\psi(x) := \phi(2x) - \phi(2x-1), \qquad x \in \mathbb{R}$$

where

$$\phi(x) = \chi_{[0,1)}(x) := \begin{cases} 1 & \text{if } x \in [0,1) \\ 0 & \text{if } x \in \mathbb{R} \setminus [0,1) \end{cases}$$

and its respective $L_2(\mathbb{R})$-orthonormal basis

$$\{\psi_{j,k}(x) = 2^{j/2}\psi(2^j x - k) : j, k \in \mathbb{Z}\}$$

were far more adept at studying the small, complicated details of Brownian motion than the trigonometric polynomials. This raises the following question: has wavelet analysis surpassed Fourier analysis, and if so, how and in what sense?

We will seek to answer this question by studying the construction and resulting properties of wavelets, comparing the wavelet series and transform to their Fourier counterparts, and by exploring the current ideas and developments within modern-day research on wavelet analysis.

Yiliu Wang, 2nd year Maths &Stats, St John's college, University of Oxford

Paradoxes in Probability

Probability is one subject that is strong in creating paradoxes. Although counterintuitive at the first sight, many in fact have significant real-world implications.

I will give two representative examples which I find most interesting in the talk. The starting point of my talk is the classic birthday problem appeared in many elementary probability textbooks. This problem can be generalized in many ways and approximated using empirical models. Then I will move to the famous St. Petersburg paradox, which alerts people of the utility of expected gain. First proposed by Daniel. Bernoulli, Karl. Menger generalized this problem to the Super-Petersburg's paradox. I will show different ways to solve this paradox, and possible variations will also be mentioned in the talk.

Paul Bowen, Coventry University

Self-avoiding walk models for polymers in solution: Monte Carlo investigation of the collapse transition of a walk model

The project was a precursor to studying the translocation of a polymer through a nano-pore at the collapse transition point. The model needed to study translocation is different from the standard model, and the location of the transition temperature needs to be found and the nature of the transition checked using the Pruned-Enriched Rosenbluth Method (PERM) Monte Carlo. This involves knowing/learning C programming to be able to understand and modify existing code as well as basic proficiency of statistical physics. Running the code and interpreting the results formed part of the project, as well as applying mathematical principles to polymers and the collapse transition point.

<u>Data Analytics and Portfolio Management</u>

Author: Uday Mohur

Institution: University of Manchester

Abstract

The focus of this paper is the application of big data with core mathematical and statistical concepts to optimise the marketing portfolio of an advertising company.

By optimal portfolio, it is implied that risk-returns analysis will be carried out for every investment channel and appropriate weightage of the marketing budget will be calculated in accordance with portfolio theory to maximise the returns while minimising the risks.

Providing a rigid mathematical definition to returns and risk is a big challenge but these parameters will be approximated by some metric, such as the click-through rate and sales revenue. Of equal importance is the correlation of the different investment channels and adequate methods will be devised to explain how they interact together. Appropriate models will then be proposed to understand the trends of investment channels for different weightages. These will then be used to create the optimal portfolio.

Other interesting aspects such as varying the marketing budget or applying other statistical ideas in terms of change-point analysis and time series will be carried out in an exploratory study to seek to briefly understand the behaviour for this multi-facetted optimisation problem.

Note: This paper only analyses high-level data. Low-level advertising analysis in terms of SEO, PPC will be solely mentioned as they fall outside the scope of this paper since they involve other crucial parameters namely semantics, behavioural economics and much more.

Abstract Word Count: 187

Sahra Ahmed Kulmiya, University of Greenwich

How has abstract mathematics impacted the discovery of the Higgs Boson?

On the 4th of July 2012, scientists at the LHC in CERN announced their discovery of a particle that was consistent with the properties that they would expect to find in the Higgs Boson. Also popularly called the God particle, the Higgs Boson is a sub-atomic particle which differs from other subatomic particles (like the electron), by being able to occupy the same point in space.

Because of this property, a bunch of Higgs Bosons can clump together to form a field. And it is this field that causes particles to have mass, for without mass, there would be nothing but free particles in the universe.

The discovery of this particle cost us $13.25 billion, where electricity costs at LHC alone cost $23.4 million.

The sheer amount of importance (both financial and scientific) dedicated this elusive particle is wondrous and fascinating. But how much of it can be credited back to its mathematical roots?

Abstract mathematics is sometimes debated to be the least applicable field of science to the real world, but this is not always the case.

For one, it furthers our intricate knowledge of the universe, which is one of the core aims of science in general.

In this presentation, I will discuss the links between abstract mathematics and theoretical particle physics, arguing for the importance of the mathematics in this field.

Joe Pollard, University of Oxford

Quantum Chaos In The Delta-Kicked Oscillator

Quantum chaos is a relatively new field of mathematics concerned with quantum systems, the classical limit of which exhibit chaotic behaviour. One notable feature of such systems is that they do not obey Ehrenfest's Theorem, which states that quantum mechanical averages follow Newton's classical equations of motion. In the case of quantum chaos, we find that there is actually a time, often called the breaking or Ehrenfest time, after which the quantum and classical predictions diverge.

In order to give an overview of the dynamics of these types of system, we present the well-studied example of the delta-kicked oscillator, which is simply the harmonic oscillator with the addition of a time-dependant kicking term. We first give a brief overview of the classical dynamics of the system, and then move on to the quantum dynamics, discussing the phenomenon of localisation and deriving an analytic expression for the breaking time.

Quantum Computing for a Quantum World

A. Kamilova[1]

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We live in a quantum world, and yet we use classical computing to represent information. Most of the time we can do it without any problems, however new quantum algorithms are becoming so complex and necessary, that we do not have computers powerful enough to perform the calculations. In 1982 Dr. Richard Feynman proposed that a computer that operates under quantum principles could reduce computational time exponentially, and since then quantum information theory has bloomed, making it a most relevant topic for today's scientific research.

In this conference we will go through some main differences between classical and quantum computations, mainly the principle of disturbance, the no cloning principle and nonlocal correlations in quantum information. Then we will proceed with a detailed mathematical representation of *qubits* or quantum bits, and some of the most elementary *quantum gates* and extend these definitions to a system of N *qubits*. We will speak briefly about Shor's algorithm, widely used in public key cryptography, which works on the factorization of very large numbers. Finally, we will discuss a phenomenon called quantum parallelism by solving Deutsch's problem with purely quantum computations, using a Hadamard quantum gate.

Thomas Wright, York University

Banach Algebras and Automatic Continuity

Traditionally, the fields of algebra and analysis have taken rather strongly opposed views of the world, the former emphasising abstraction and structural results, and the later focusing on the properties of specific sequences, series, functions, and equations. However, in the late 19th and early 20th century, ideas from abstract algebra increasingly began first to influence and then to revolutionise other areas of mathematics as diverse as number theory, geometry, and mathematical physics. Analysis was no exception, and I aim to tell the story of how Banach algebras bring the two subjects together, combining an algebraic structure (an algebra) with an analytical one (a Banach space), and look into Automatic Continuity Theory, a subject which explores when analysis is actually algebra in disguise, that is, when is a Banach algebra completely determined by its algebraic properties (up to algebraic and topological isomorphism)?

Cameron Whitehead, University of Oxford

D-modules on singular schemes

A D-module on a scheme can be seen as the algebraic version of a manifold with flat connection, and studying these objects is very profitable for smooth schemes. However, for singular schemes, the category of D-modules can be very badly behaved, since the sheaf of differential operators need not be noetherian or even finitely generated. So working with this category is particularly hard, and a different approach to the geometry of schemes is needed. This alternative concept is provided by the D-crystals developed by Grothendieck, which in some sense interpret the idea of parallel transport in an algebraic setting.

In this talk we will define the category of crystals on a not necessarily smooth scheme, and see an equivalence of categories which shows us that these crystals really are an reinterpretation of the concept of a D-module from the smooth to singular setting. Specifically, we will see that on a smooth scheme the categories of crystals and of D-modules are equivalent, and for all underlying schemes the category of crystals is locally noetherian.

Why Antarctica seems so big

Henrique Rui Neves Aguiar, University of Oxford

Mapping the earth has been of critical importance for Men ever since the dawn of first civilizations. It was, however, only much later that the required mathematics began to be developed. Modeling the earth as a sphere, in what ways can we represent it in 2 dimensions so that angles, distances and areas are preserved? Is there a map of the world we can accurately use to move from one city to another?
I will talk about the specific case of cylindrical projection, a very common type of such representations, which satisfy the equations $x(\theta, \lambda) = R\lambda$ and $y(\theta, \lambda) = Rf(\theta)$. I will then deduce formulas for lengths, areas and angles in these projections, and speak about the most famous cylindrical projections, namely, The Equidistant projection ($f(\theta) = \theta$), Lamberts equal-area projection ($f(\theta) = sin(\theta)$), Mercators projection ($f(\theta) = ln(tan(\theta/2 + \pi/4))$) and the Central cylindrical projection ($f(\theta) = tan(\theta)$). For each of these, I am going to provide details of their advantages and disadvantages, and conclude with some common misconceptions most of us possess regarding the countries of our own world.

Classifying Tiling Spaces of the same Cohomology

Robert Findlay

Mathematics, University of Durham

Aperiodic tilings first received widespread interest after a fateful article by Martin Gardener on the Penrose tiling. These types of tilings are interesting because of the symmetries allowed that would otherwise be impossible, they have found applications in Quasicrystals which have been used in strange new alloys in non-stick pans and other areas where special steels are needed.

The topology of tilling spaces is a concept that allows us to study the properties of aperiodic tilings using methods coming from Algebraic Topology. Since Anderson and Putnam's work in the 1990s the methods used to differentiate between tilings have mostly relied on Cohomolgy. However more recently an invariant created by Hunton and Clark allows us to determine if a 1 dimensional substitution tilling can be embedded in a surface. This property is not distinguishable by Cohomology.

This talk will focus on the method of taking 1-dimensional tiling spaces of the same Cohomology, classifying them based on the Hunton-Clark L-invariant using Stallings Folding and then trying to further differentiate between them while not assuming any knowledge of homological algebra or shape theory. This will lead us to interesting results about the existence of embeddings of any tiling spaces.

Chan Bae, University of Oxford

Embedding graphs in $\mathbf{R}^3$ *without self-intersections*

Kuratowski's theorem tells us exactly what graphs can and can't be drawn in the plane without self-intersections. A natural question arises: What about $\mathbf{R}^3$? We will find that any finite graph can be drawn in space without self-intersections. How much do we have to distort the graph to eliminate self-intersections? What happens if we make the graph infinite? And what does the continuum hypothesis have to do with any of this?

Frobenius algebras and cobordism diagrams

Vitalijs Brejevs[1]

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An algebra over a field is a vector space on which bilinear multiplication is defined. A related definition is one of a coalgebra, obtained by 'reversing the arrows' in the commutative diagrams for an algebra. The focus of the talk would be on the notion of a Frobenius algebra — an algebra with an additional structure which simultaneously makes it into a coalgebra in a natural way.

One way to prove some interesting results about Frobenius algebras is to use the pictorial language of cobordism diagrams employed in topology. By manipulating topological objects like 'pairs of pants' one can gain insight in the structure of Frobenius algebras without using rather cumbersome notation. The main aims of the talk would be to give the necessary basic definitions, establish graphical calculus on algebras and coalgebras and use it to give an explicit construction of the unique coalgebra structure on any given Frobenius algebra.

Robert Long, Coventry University

Numerical studies of electro-optic properties of PbSe Nano-crystals

Semiconductor nanostructures have numerous potential applications because their optical and electronic properties can be readily tuned by varying their size, shape, structure and composition. The range of optoelectronic applications that can utilise nanoparticles include photovoltaic power production and nanoscale electronics. The surface and shape of the nanostructures play a crucial role for their electro-optic properties.

We report systematic investigations of the electrostatic and structural trends of PbSe nanostructures using empirical potentials, dynamical simulations, time-dependent density functional theory and non-linear regime. Numerical Simulations were carried out by a powerful computing tool CASTEP which makes use of density functional theory to calculate the electronic properties of crystalline structures from first principles. The primary calculation in CASTEP is that of the product of the Hamiltonian and a wave function. Fast Fourier Transforms are used to transform quantities such as wave functions, and potentials from real to reciprocal space, and vice versa. CASTEP employs the finite difference method applying it to repeated calculations on a series of systems; this allows quantum mechanical calculations to be carried out with both high efficiency and accuracy.

We find significant differences in the optical properties of PbSe nanostructures depending on size, shape, and structure.

Andrew Kelly, Birkbeck, University of London

Solving commercial colour-rending problems of LED illumination using multi-die sources and linear programming

We will discuss the problem of achieving a desired colour from multiple LED-dies by first discussing the problem of colour rendering in illumination. It will be shown that plain white or RGB-only LED lighting is insufficient for illuminating a surface sufficiently to allow any colour to be authentically displayed, necessitating the introduction of multiple LED sources. This leads to a problem which is the core topic of this talk: How do we set n-source LED dies to achieve a target colour?

Starting with the tristimulus vector representations of colour, we will go on to show that the solution is the intersection of three hyperplanes in $\mathbf{R}^n$. Concepts in colour science such as the CIE colour locus and MacAdam's ellipse will be used to relax the equality constraints to convert the solution space to the intersection of six closed convex halfspaces in $\mathbf{R}^n$. A coordinate transform will be shown which changes MacAdam's ellipses on the CIE locus to standard circles, converting the problem into a linear system, which can be used to minimise an objective function, which will be the total power draw. This is now a linear program, which can be solved by the Simplex Method.

David Paintin, University of East Anglia

The Continuum Hypothesis

I would like to talk about the continuum hypothesis and outline some for mathematicians of any background of the methods that were used to prove its independence from ZF.

Cantor proved in 1874 that the reals were uncountable, and 17 years later published his famous diagonal argument, a concise and elegant way of showing that the reals are uncountable. Though Cantor was unable to gain much traction immediately, Cantor's ideas would spark the imagination of various mathematicians, and would lead to the development of set theory. While this was able to provide good insight, a fundamental question still remained. Are there sets of reals with cardinality in between that of the natural numbers and that of the reals? This problem would take nearly 100 years to find a solution, and would lead to many developments within set theory.

I would like to outline the notions needed to understand the continuum hypothesis and the key concepts used by Gödel and Cohen to solve it. This is a rich topic that intertwines history and mathematics, while also exposing undergraduates to an important part of modern mathematics which they may not have seen.

Infinity is Weird: The Axiom of Choice And You

And You And You And You And You And You And You And You And You...

Jacob Bennett-Woolf

December 7, 2015

Can we well-order every set? This question, in 1904, led Ernst Zermelo to the first formal definition of the Axiom of Choice. The Axiom itself seems innocuous enough, that every set has a choice function, however it has many far reaching implications, such as non-linear solutions to the equations $f(x+y) = f(x)+f(y)$ and paradoxical decompositions of the sphere, the Banach-Tarski paradox. One hundred and eleven years after it was first formally defined, it is now implicit in almost all areas of Pure Mathematics.

The proposed talk will begin with a brief tour of many different topics related to the Axiom of Choice, ranging from defining the sizes of infinite sets, to the philosophy of the Axiom. Once we have completed this foray, content of the rest of the talk shall be opened up to the audience, allowing them to decide the area they wish to hear more about.

Theory of Distributions and Linear Differential Equations

Abstract

Ruoyu Wang*

December 10, 2015

The theory of distributions, also named the theory of generalised functions, was well developed by Laurent Schwartz in 1940s. The theory is a generalisation of classical analysis, which provides a wider framework in which we can reformulate and develop classical problems in a perspicuous language. Its main idea is to view functions as continuous linear functionals acting on the space of 'well-behaved' functions. In this sense of extension, we can define some functions which are irregular, for example the Dirac delta function, and even derivatives at discontinuities. This allows us to find a reasonable explanation to non-classical solutions to PDEs, which are not smooth enough in the classical sense, and moreover provides a general approach to solve linear PDEs.

The talk will quickly go through the following points: i) definitions and basic operations; ii) irregular distributions; iii) derivatives at discontinuities; iv) examples of solving linear ODEs and PDEs by the theory; v) short comments on weak solutions to the Navier-Stokes equations.

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Rasa Giniunaite, University of Warwick

Abstract

Numerical schemes for geometric PDEs

The analysis of geometric PDEs is mainly driven by advances in techniques in cell biology, imaging and data acquisition. I have analysed and compared two finite element schemes for a system consisting of a geometric evolution equation for a curve and a parabolic equation on the evolving curve. More precisely, curve shortening flow with a forcing term that depends on a field defined on the curve is coupled with a diffusion equation on a field. The first one was simply derived by formulating a weak problem and applying finite element method whereas the second also used the the reparametrization using the idea from the Ricci flow known as the DeTurck trick. I worked on the error estimates for a semi-discrete schemes in both cases. The numerical simulations of the fully-discrete schemes showed that the mesh properties were significantly improved in the reparametrized scheme.

Lizhi Zhang, University of Strathclyde

The stability of classic predator-prey models and modification

Predator-prey relationships are a pillar of ecological modelling. Focusing on this interesting and wide extended interaction link, mathematicians successfully apply differential equations to describe how predator and prey populations change through time. The steady solutions of the system of equations correspond to the ecological balance, with constant relative values of the population densities in the system. A classic example is the Lotka-Volterra equations, which are a pair of first-order non-linear, differential equations frequently used to describe the dynamics of this kind of ecological system.

However, many oversimplified assumptions in this and other classic models lead to misleading results, which may burden our predictions. On the other hand, additional layers of complexity may lead to unstable models or difficult to parameterize.

In this talk, we will discuss how an increasing level of realism affects the stability of the predator-prey system of equations. Starting from simple, classic model, I will show how the addition of new terms describing, e.g., predator physiology (satiation) normally contributes to the instability of the system. Also, I will show how these more realistic ingredients contribute to reaching steady state faster (when they exist). Our results can potentially contribute to developing new predator-prey models balancing simplicity and realism.

Topic: Comparative Study of Two Epidemic Models on Realistic Networks

Bin Wang

This project deals with different epidemic models including the susceptible-infected-susceptible (SIS) model [1] and the susceptible-infected-removed (SIR) model [2] [3]. Such models can be applied to real networks such as Internet [4] [5] [6], disease spreading and human sexual contacts [7], etc. All these networks share a bounded scale-free behaviour due to their limited size. Using epidemic models and these networks, we can learn the characteristics of the epidemic in realistic examples.

A SIS model will be constructed, for which we will define the so-called epidemic threshold λ_c that determines whether the epidemic spreads [8]. The first goal of the study is to investigate the properties of λ_c. In addition, the relative density $\rho_k(t)$, $i.e.$ the probability that a node is infected when it is linked by k edges, will also play an important role. The next step will be considering the SIR model. Immunization will be possible and results in a different pattern of infection spreading. We will follow similar steps to understand how infection spreads when immune individuals are present in the system.

We will then compare both SIS and SIR. Putting theory and reality together can improve our knowledge of both aspects of the problem.

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